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proper attention must be paid to the degree of condensation of the objects. But there is an obvious way of avoiding systematic errors altogether (except such as are well within the limit of accuracy attainable in such observations) by giving up observations of transits and measuring both $\Delta\alpha$ and $\Delta\delta$ by the micrometer screw. As this can only be done by selecting comparison stars among those visible in the same field with each nebula, and such stars in many cases cannot be observed on the meridian, observers will have to sacrifice the pleasure of putting together a list of Right Ascensions and Declinations of the nebulæ they have measured. But this small sacrifice will be well compensated by the avoidance of the difficulty of assigning to each object the proper amount of systematic error.

Armagh Observatory:
1896 *December.*

On the Inequality in the Apparent Diurnal Movement of Stars due to refraction, and a method of allowing for it in Astronomical Photography. By Prof. Arthur A. Rambaut, M.A., D.Sc.

It is obvious that the varying effect of refraction in the Earth's atmosphere to which the light of a star is subjected as its hour-angle changes will cause inequalities in its apparent diurnal movement, and that no uniform equatorial movement will enable a telescope to follow a star for long periods of time when the highest accuracy is aimed at.

The irregularities in the apparent motion of stars due to local and temporary changes in the refraction are well known to astronomers who have had experience in stellar photography, but I doubt whether the perfectly regular and systematic change as the star alters its position in the sky has received the attention on the part of astronomers which its importance seems to demand.

Speaking generally, the hour-angle of a star is diminished by the refraction when approaching its upper culmination, but to a continually decreasing extent, and consequently the motion of the star at this part of its course appears slower than it actually is. At first sight it might perhaps appear that the effect at the western side of the meridian would be of an exactly opposite kind, and that the stars at this part of their diurnal track would appear to move more rapidly than they actually do. But a moment's reflection will show that this is not the case, for the refraction throwing the apparent to the following side of the true image in this case, and the displacement continually increasing as the star sinks in the west, the apparent will fall ever more and more behind the true position.

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Now this inequality in the apparent movement occasions a great part of the difficulty in making a telescope follow accurately for long periods of time, and necessitates most of the watching and guiding by hand which renders a long exposure so laborious and fatiguing. The equatorial movement of the instrument being a uniform one, it is impossible to rate an equatorial clock so as to follow a star with equal accuracy at all parts of its course. The changes in the refraction are, indeed, from one point of view so slow that in most cases the clock can be rated to follow a particular star with sensible accuracy for reasonable intervals of time; but no adjustment will suit all stars equally well, and to obtain the best results the rate of the clock ought in each case to be adjusted for the particular star under consideration, and if the exposure is prolonged the rate ought to be suitably altered from time to time.

As an example, I may state here, in order to show that the differences to which I refer are not negligible quantities, that if a clock were rated by the motion of a star at the zenith in these latitudes, it would gain more than one-thirtieth of a second of time each minute on a star crossing the meridian at declination $77\frac{1}{2}°$, which would cause the star to fall behind the cross wires at the rate of one-tenth of a second of *arc* per minute; a rate of displacement which would very soon make itself sensible in any telescope of moderate dimensions. At a distance from the meridian the differences would be greater; and if the exposure were continued for, say, four hours, it would require frequent hand correction, amounting in the aggregate to more than 24 seconds, to counteract its effect.

It will thus be seen that the varying effect of refraction at different parts of the sky upon the motion of a star is quite noticeable, and that if this disturbance could be removed the art of steering a photo-telescope so as to point continuously at a given star, which, under the most favourable circumstances, is no easy task, might be rendered much less exacting and severe.

My attention has been specially directed to this question by the fact that in the class of work to which I have devoted our telescope at the Observatory of Trinity College, Dublin, viz., the measurement of stellar parallax, it is often necessary to take photographs of star groups at considerable hour-angles when the effect of refraction is most noticeable. If the observer's object is merely to obtain pictures of star groups, the work can be so arranged that each object is photographed when it arrives at or near the meridian. Thus almost all Dr. Roberts' photographs were taken at small hour-angles. The photographs for the international photographic survey of the heavens are also taken close to the meridian, where the variation in the angular movement of stars is least felt.

It is different, however, when it is intended to utilise the plates for the detection of stellar parallax. In connection with this research, it is desirable that a large proportion of the photographs

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of each object should be taken when the star is near the apses of its parallactic ellipse, and this condition often necessitates photographing a star at hour-angles of six hours or more.

I have thus been led to examine minutely the effect of refraction on the movement of a star; and since a method which enables good photographs to be taken comparatively easily at any altitude, provided the atmosphere is steady enough for good seeing, cannot fail to be useful, perhaps a description of the means I have been led to adopt for this purpose may prove of interest to many who are engaged in this or other branches of stellar photography.

Let us examine what the effect of refraction is. If we denote the apparent western hour-angle of a star by h, the effect of refraction in hour-angle by Δh, the right ascension by α, and the sidereal time by θ, we have

$$h = \theta - \alpha + \Delta h,$$

$$\frac{dh}{d\theta} = 1 + \frac{d\Delta h}{d\theta}.$$

Hence the expression $\frac{d\Delta h}{d\theta}$ measures the rate at which the apparent movement of a star gains on its mean movement or on sidereal time.

To calculate this quantity we have the equation:

$$\left.\begin{array}{ll} & \sin\eta \cot\Delta h = \cot\Delta\zeta \cos\delta - \sin\delta\cos\eta \\ \text{Also} & \sin(\delta + \Delta\delta) = \sin\delta\cos\Delta\zeta + \cos\delta\sin\Delta\zeta\cos\eta \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (\alpha)$$

in which δ denotes the declination, ζ the zenith distance, and η the parallactic angle, of the star, and $\Delta\zeta$, Δh, and $\Delta\delta$ are the effects of refraction in ζ, h, and δ respectively. For our present purpose we may put $\Delta\zeta = \beta \tan\zeta$, β being a function of ζ and so small that we may neglect its square and higher powers.

Hence

$$\Delta h = \beta \tan\zeta \sin\eta \sec\delta = \beta . \psi, \text{ say.}$$

Differentiating, we find

$$\frac{d\Delta h}{a\theta} = \beta \frac{d\psi}{d\theta} + \psi \frac{d\beta}{d\theta}.$$

Except in the immediate vicinity of the horizon, the second term in this may be neglected,* and we get, therefore,

$$\frac{dh}{d\theta} = 1 + \beta \frac{d\psi}{d\theta}.$$

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* See a paper in the *Astronomische Nachrichten*, No. 3,125, on "The Corrections for Refraction to Measures of Stellar Photographs."

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Also by the ordinary formulæ of transformation, if ϕ denotes the latitude of the observatory, we have

$$\psi = -\frac{\cos\phi}{\cos\delta}\cdot\frac{\sin h}{\sin\delta\sin\phi+\cos\delta\cos\phi\cos h};$$

and therefore

$$\frac{dh}{d\theta} = 1-\beta\,\frac{\cos\phi}{\cos\delta}\cdot\frac{\cos\delta\cos\phi+\sin\delta\sin\phi\cos h}{\{\sin\delta\sin\phi+\cos\delta\cos\phi\cos h\}^2}.$$

Assuming m, n, μ, ν such that

$$\tan m=\cot\phi\cos h \qquad \cot\mu=\tan\phi\cos h$$

$$\cot n=\sin m\tan h \qquad \cot\nu=\cos\mu\tan h$$

this may be written in the form

$$\frac{dh}{d\theta}=1-\beta\,\frac{\cos\phi}{\cos\delta}\cdot\frac{\sin\nu\sin(\mu+\delta)}{\sin^2 n\sin^2(m+\delta)} \quad . \; . \; . \; . \; . \; . \qquad (\beta)$$

If it were required that the telescope should follow the star with absolute precision, it would be necessary to construct a clockwork system which would drive the instrument at a rate varying continually with the hour-angle according to the law expressed by this formula (β). In practice, however, it is sufficient to alter the rate at intervals, the length of which will depend upon the rapidity of the changes in the refraction, provided always that the error thus introduced does not exceed a certain definite limit.

From the formula (β) I have computed the values of $\frac{dh}{d\theta}-1$ for every tenth degree in declination, and for every even hour in hour-angle, and from the quantities thus obtained I have drawn the series of curves in fig. 1, in which the abscissæ denote the gaining or losing rate of the star as compared with sidereal time, expressed in the usual way as so many seconds a day.

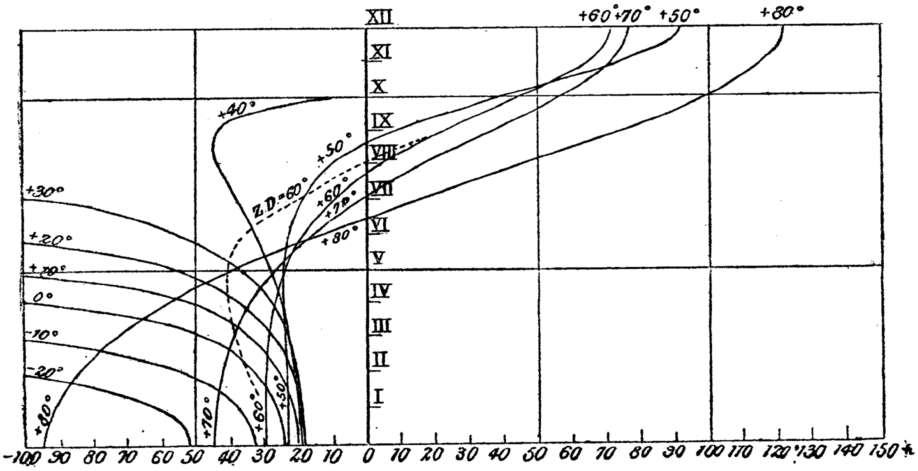

Fig. 1.

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Now an examination of these curves shows that, even when the change in the rate of movement is most rapid, a uniform rate, *if suitably chosen*, will not in ten minutes introduce an error amounting to the one-twentieth of a second, which is about the limit within which the accidental errors of a good equatorial clock can be kept. This, then, would appear to be the limit of time for which, in cases of extremely rapid changes in the stars' movements, a uniform rate may be used. In most cases a longer interval is allowable.

If the exposure is prolonged beyond the period for which a uniform rate is permissible, it will be necessary to alter the rate from time to time in such a way that the algebraic sum of the errors thus introduced, alternately in excess and defect, shall be always less than the assumed limit of accuracy.

This is particularly easy to manage with a clock controlled by the action of a pendulum. By employing a series of weights the effect of which on the period of oscillation of the pendulum has been ascertained, and by suitably altering the weights upon the shelf of the pendulum from time to time as the diagram shows to be necessary, the movement of the clock, and consequently that of the telescope, can be made to agree with the apparent rate of movement of the star to any degree of precision required.

The length of exposure for which a uniform rate is permissible is determined in the following way from the curve drawn, as in fig. 1, for the declination of the star :—

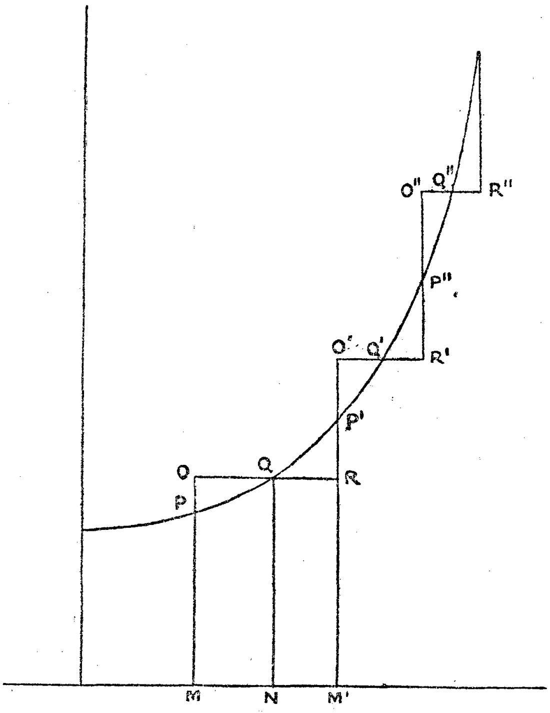

Fig. 2.

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If r is the variable rate at which the star gains on the mean sidereal movement as given by formula (β), then the total gain in the interval $t'-t$ is $\int_t^{t'} r dt$.

If for the same interval of time we take a uniform rate r', the amount which the motion of the telescope gains on mean sidereal movement is $r'(t'-t)$, and the amount by which the telescope gains upon the star is represented by the difference between these two expressions. But the first is in fig. 2 represented by the area PP'M'M under the curve, while the second is the parallelogram ORM'M. Hence the displacement of the telescope with regard to the star at the end of the exposure is measured by the difference of the two areas OPQ and QRP', and by taking a uniform rate represented by QN, corresponding to about the middle of the exposure, it can be used for the whole interval MM' provided that neither area OPQ nor QRP' exceeds the assumed limit of accuracy.

On the scale I have used, viz., 1^{mm} in the abscissæ to represent a rate of 1^{s} per day and 10^{mm} in the ordinates to represent an hour, this area, corresponding to a gain or loss of one-twentieth of a second, is 12 square millimetres, which at once enables us to judge for what length of exposure we may venture to use a uniform rate.

By drawing a zigzag line such that the approximately triangular areas contained between it and the curve shall never

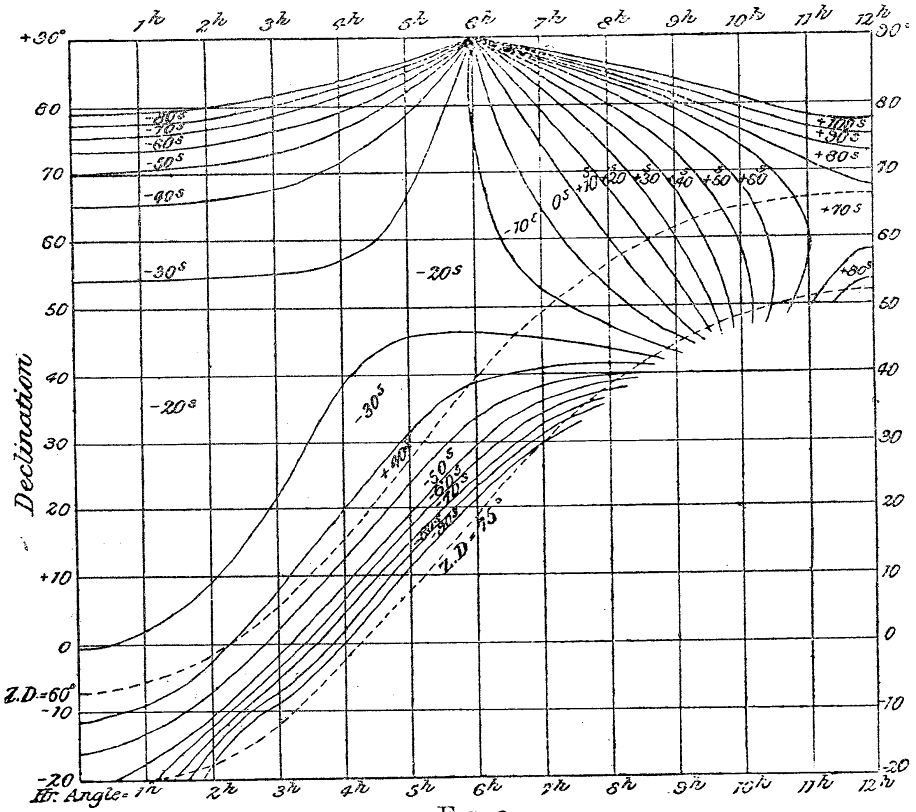

Fig. 3.

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exceed 12 square millimetres, we get a series of uniform rates which will keep the pointing of the telescope constantly within one-twentieth of a second of the star.

When only short exposures of 5 or 10 minutes are necessary, as is generally the case with plates intended for accurate measurement, in which it is advisable to keep the star discs as small as possible, a still simpler method is available. By plotting down on squared paper the points on the sky for which the appropriate rates are -105^s, -95^s, . . . -5^s, $+5^s$, . . . $+95^s$, $+100^s$ a day, the abscissæ representing hour-angles and the ordinates representing declinations, we get a series of curves the spaces between which represent the regions of the sky for which the appropriate rates are -100^s, -90^s, . . . 0^s, . . . $+90^s$, $+100^s$ a day, and thus by a simple reference to this diagram, fig. 3, it is possible to read off the suitable clock-rate with sufficient accuracy.

This method I have found most convenient in practice for short exposures. I can turn the telescope with the greatest confidence from a star at its upper culmination, to follow which a star must lose at the rate of from 18 to 100 seconds or more a day, according to the declination, to another at its lower culmination apparently gaining at the rate of 70 or 80 seconds a day, and I find the telescope will follow both with equal accuracy.

The following table contains the rates suitable to each degree of declination, interpolated from the computed values referred to above, for the latitude of Dublin (53° 23′ 13″), and will serve to show through what a wide range the apparent angular velocity of a star varies. In this table + denotes a gaining, and — a losing, rate as compared with mean sidereal time.

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TABLE.

Hour Angle.

Decl.	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h
°	s	s	s	s	s	s	s	s	s	s	s	s	s
−20	−52·0	−61·2	−104·1	...	...	...	...	...	...	...	...	...	...
19	−49·4	−58·0	− 97·4	...	...	...	...	...	...	...	...	...	...
18	−46·9	−54·8	− 91·1	...	...	...	...	...	...	...	...	...	...
17	−44·6	−52·0	− 85·3	...	...	...	...	...	...	...	...	...	...
16	−42·5	−49·3	− 79·8	...	...	...	...	...	...	...	...	...	...
15	−40·5	−46·8	− 74·6	...	...	...	...	...	...	...	...	...	...
14	−38·6	−44·6	− 69·8	...	...	...	...	...	...	...	...	...	...
13	−36·9	−42·4	− 65·4	...	...	...	...	...	...	...	...	...	...
12	−35·4	−40·4	− 61·2	...	...	...	...	...	...	...	...	...	...
11	−33·9	−38·5	− 57·5	...	...	...	...	...	...	...	...	...	...
10	−32·7	−36·8	− 54·0	−110·1	...	...	...	...	...	...	...	...	...
9	−31·5	−35·3	− 50·9	−100·7	...	...	...	...	...	...	...	...	...
8	−30·4	−34·0	− 48·1	− 92·2	...	...	...	...	...	...	...	...	...
7	−29·4	−32·8	− 45·5	− 84·3	...	...	...	...	...	...	...	...	...
6	−28·6	−31·6	− 42·0	− 77·2	...	...	...	...	...	...	...	...	...
5	−27·8	−30·5	− 40·8	− 70·9	...	...	...	...	...	...	...	...	...
4	−27·0	−29·5	− 38·7	− 65·2	...	...	...	...	...	...	...	...	...
3	−26·2	−28·5	− 36·8	− 60·1	...	...	...	...	...	...	...	...	...
2	−25·6	−27·6	− 35·2	− 55·6	...	...	...	...	...	...	...	...	...

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Decl.	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h
°	s	s	s	s	s	s	s	s	s	s	s	s	s
−1	−24·9	−26·8	−33·7	−51·5	...	...	...	...	...	...	...	...	...
0	−24·3	−26·1	−32·3	−48·1	−94·0	...	...	...	...	...	...	...	...
+1	−23·8	−25·5	−31·3	−46·1	−88·0	...	...	...	...	...	...	...	...
2	−23·3	−25·0	−30·4	−44·1	−82·5	...	...	...	...	...	...	...	...
3	−22·9	−24·4	−29·6	−42·3	−77·3	...	...	...	...	...	...	...	...
4	−22·4	−23·9	−28·8	−40·7	−72·5	...	...	...	...	...	...	...	...
5	−22·0	−23·4	−28·0	−39·1	−68·2	...	...	...	...	...	...	...	...
6	−21·6	−23·0	−27·3	−37·6	−64·1	...	...	...	...	...	...	...	...
7	−21·3	−22·5	−26·6	−36·3	−60·4	...	...	...	...	...	...	...	...
8	−20·9	−22·1	−26·0	−35·1	−57·0	...	...	...	...	...	...	...	...
9	−20·6	−21·8	−25·4	−32·9	−53·9	...	...	...	...	...	...	...	...
10	−20·3	−21·4	−24·9	−32·8	−51·1	−104·2	...	...	...	...	...	...	...
11	−20·0	−21·1	−24·5	−31·9	−48·7	−97·2	...	...	...	...	...	...	...
12	−19·8	−20·8	−24·0	−31·0	−46·8	−90·8	...	...	...	...	...	...	...
13	−19·6	−20·5	−23·6	−30·2	−45·0	−84·8	...	...	...	...	...	...	...
14	−19·4	−20·3	−23·2	−29·4	−43·2	−79·4	...	...	...	...	...	...	...
15	−19·2	−20·1	−22·9	−28·7	−41·6	−74·3	...	...	...	...	...	...	...
16	−19·1	−19·9	−22·6	−28·1	−40·0	−69·6	...	...	...	...	...	...	...
17	−18·9	−19·7	−22·3	−27·5	−38·6	−65·4	...	...	...	...	...	...	...
18	−18·8	−19·5	−22·0	−26·9	−37·2	−61·6	...	...	...	...	...	...	...
19	−18·7	−19·3	−21·8	−26·4	−36·0	−58·0	...	...	...	...	...	...	...

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Decl.	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h
°	s	s	s	s	s	s	s	s	s	s	s	s	s
20	−18·6	−19·2	−21·7	−25·9	−34·9	−54·7	−110·0	…	…	…	…	…	…
21	−18·5	−19·1	−21·5	−25·5	−34·0	−52·2	−102·0	…	…	…	…	…	…
22	−18·5	−19·0	−21·3	−25·1	−33·1	−49·9	−94·5	…	…	…	…	…	…
23	−18·4	−18·9	−21·1	−24·8	−32·2	−47·8	−87·6	…	…	…	…	…	…
24	−18·4	−18·8	−21·0	−24·4	−31·4	−45·8	−81·2	…	…	…	…	…	…
25	−18·3	−18·8	−20·9	−24·1	−30·7	−43·9	−75·3	…	…	…	…	…	…
26	−18·3	−18·8	−20·8	−23·8	−30·0	−42·1	−69·9	…	…	…	…	…	…
27	−18·3	−18·7	−20·7	−23·6	−29·4	−40·5	−65·1	…	…	…	…	…	…
28	−18·3	−18·8	−20·6	−23·3	−28·8	−38·9	−60·7	…	…	…	…	…	…
29	−18·4	−18·8	−20·4	−23·1	−28·2	−37·5	−56·6	…	…	…	…	…	…
30	−18·4	−18·8	−20·2	−22·9	−27·7	−36·2	−52·9	−92·2	…	…	…	…	…
31	−18·4	−18·8	−20·2	−22·7	−27·3	−35·1	−49·9	−84·5	…	…	…	…	…
32	−18·5	−18·9	−20·3	−22·6	−26·9	−34·0	−47·4	−77·4	…	…	…	…	…
33	−18·7	−18·9	−20·3	−22·4	−26·5	−33·1	−45·2	−70·8	…	…	…	…	…
34	−18·8	−19·0	−20·3	−22·3	−26·1	−32·2	−43·0	−64·8	…	…	…	…	…
35	−18·9	−19·2	−20·3	−22·2	−25·8	−31·3	−40·8	−59·2	…	…	…	…	…
36	−19·0	−19·3	−20·4	−22·2	−25·5	−30·5	−38·8	−54·1	…	…	…	…	…
37	−19·1	−19·4	−20·4	−22·2	−25·2	−29·7	−37·0	−49·4	…	…	…	…	…
38	−19·2	−19·6	−20·4	−22·2	−24·9	−29·0	−35·3	−45·3	…	…	…	…	…
39	−19·4	−19·7	−20·5	−22·2	−24·7	−28·4	−33·8	−41·5	…	…	…	…	…
40	−19·6	−19·9	−20·7	−22·2	−24·5	−27·8	−32·3	−38·1	−43·9	−42·8	…	…	…

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Decl.	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h
°	s	s	s	s	s	s	s	s	s	s	s	s	s
41	−19·8	−20·1	−20·8	−22·2	−24·3	−27·3	−31·1	−35·6	−39·1	−34·8	...	...	...
42	−20·1	−20·3	−21·0	−22·3	−24·1	−26·9	−29·9	−33·2	−34·5	−27·4	...	...	...
43	−20·5	−20·6	−21·2	−22·4	−24·0	−26·4	−28·9	−30·9	−30·4	−20·8	...	...	...
44	−20·8	−20·9	−21·5	−22·6	−23·9	−26·0	−27·9	−28·8	−26·5	−14·8	...	...	...
45	−21·1	−21·2	−21·7	−22·7	−23·8	−25·6	−26·9	−26·8	−23·0	−9·5	...	...	...
46	−21·4	−21·5	−22·0	−22·8	−23·8	−25·3	−26·0	−25·1	−19·8	−4·7	...	...	...
47	−21·7	−21·8	−22·2	−23·0	−23·8	−24·9	−25·1	−23·4	−16·9	−0·4	...	...	...
48	−22·1	−22·1	−22·5	−23·2	−23·9	−24·6	−24·3	−21·8	−14·1	+3·4	...	..	...
49	−22·4	−22·5	−22·8	−23·4	−24·0	−24·3	−23·6	−20·3	−11·9	+6·7	...	...	...
50	−22·8	−22·9	−23·2	−23·6	−24·1	−24·1	−22·9	−18·9	−9·4	+9·6	...	...	...
51	−23·3	−23·3	−23·7	−23·9	−24·2	−23·9	−22·3	−17·5	−7·4	+11·8	+41·3	+73·9	+89·0
52	−23·8	−23·8	−24·1	−24·3	−24·4	−23·7	−21·7	−16·3	−5·6	+13·6	+42·2	+72·6	+86·5
53	−24·3	−24·3	−24·6	−24·7	−24·5	−23·5	−21·1	−15·2	−3·9	+15·3	+42·9	+71·4	+84·1
54	−24·9	−25·0	−25·0	−25·0	−24·7	−23·4	−20·5	−14·2	−2·4	+16·8	+43·4	+70·2	+81·8
55	−25·6	−25·6	−25·5	−25·4	−25·0	−23·3	−20·0	−13·2	−1·1	+18·3	+43·8	+69·0	+79·7
56	−26·3	−26·2	−26·1	−25·8	−25·2	−23·2	−19·6	−12·3	+0·2	+19·5	+44·4	+67·8	+77·8
57	−27·0	−26·9	−26·8	−26·4	−25·4	−23·1	−19·2	−11·4	+1·5	+20·7	+44·7	+66·8	+76·0
58	−27·7	−27·6	−27·5	−27·0	−25·7	−23·1	−18·8	−10·6	+2·6	+21·8	+45·0	+65·9	+74·4
59	−28·5	−28·4	−28·2	−27·6	−26·1	−23·2	−18·4	−9·8	+3·7	+22·8	+45·3	+65·1	+73·1
60	−29·4	−29·3	−28·9	−28·1	−26·4	−23·3	−18·0	−9·0	+4·8	+23·8	+45·7	+64·5	+72·0
61	−30·4	−30·3	−29·8	−28·8	−26·7	−23·4	−17·6	−8·2	+5·9	+24·6	+46·1	+64·0	+71·0

Decl.	0h	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h
°	s	s	s	s	s	s	s	s	s	s	s	s	s
62	−31·5	−31·3	−30·9	−29·7	−27·4	−23·5	−17·2	−7·5	+6·8	+25·3	+46·5	+63·7	+70·4
63	−32·8	−32·6	−32·0	−30·6	−28·0	−23·7	−16·9	−6·7	+7·9	+26·3	+47·1	+63·7	+70·1
64	−34·1	−33·9	−33·1	−31·5	−28·7	−23·9	−16·5	−5·9	+9·0	+27·3	+47·9	+64·0	+70·1
65	−35·6	−35·4	−34·3	−32·5	−29·4	−24·2	−16·2	−5·2	+10·1	+28·4	+48·9	+64·5	+70·5
66	−37·2	−36·9	−35·7	−33·6	−30·1	−24·6	−16·0	−4·4	+11·3	+29·7	+50·2	+65·4	+71·0
67	−38·8	−38·5	−37·2	−34·8	−30·9	−24·9	−15·8	−3·6	+12·6	+31·2	+51·6	+66·5	+72·0
68	−40·6	−40·2	−38·8	−36·1	−31·8	−25·2	−15·6	−2·7	+14·0	+33·0	+53·2	+67·9	+73·3
69	−42·5	−42·0	−40·5	−37·5	−32·8	−25·6	−15·4	−1·8	+15·5	+35·1	+55·0	+69·6	+74·9
70	−44·5	−44·0	−42·3	−39·0	−33·8	−26·1	−15·3	−0·8	+17·3	+37·5	+57·0	+71·5	+76·9
71	−47·4	−46·9	−44·9	−41·2	−35·4	−26·8	−15·1	+0·2	+19·0	+39·9	+59·4	+73·9	+79·3
72	−50·9	−50·2	−47·8	−43·5	−37·1	−27·6	−14·9	+1·4	+21·0	+42·3	+62·1	+76·8	+82·2
73	−54·5	−53·8	−51·2	−46·3	−38·9	−28·6	−14·7	+2·7	+23·1	+45·0	+65·2	+80·0	+85·5
74	−58·7	−57·8	−54·8	−49·2	−41·1	−29·7	−14·5	+4·0	+25·5	+48·1	+68·7	+83·7	+89·2
75	−63·3	−62·2	−58·8	−52·6	−43·5	−30·9	−14·4	+5·4	+28·1	+51·5	+72·7	+87·8	+93·4
76	−68·4	−67·1	−63·2	−56·2	−46·1	−32·1	−14·3	+7·0	+30·9	+55·2	+77·1	+92·3	+98·1
77	−74·0	−72·5	−68·1	−60·2	−48·9	−33·5	−14·2	+8·6	+33·9	+59·4	+81·8	+97·5	+103·3
78	−80·2	−78·5	−73·4	−64·6	−52·0	−35·1	−14·1	+10·3	+37·2	+64·0	+87·0	+103·1	+109·0
79	−86·8	−84·9	−79·3	−69·4	−55·3	−36·8	−14·0	+12·3	+40·8	+68·8	+92·8	+109·2	+115·2
80	−94·0	−91·9	−85·5	−74·6	−59·0	−38·7	−13·9	+14·5	+44·8	+74·2	+99·1	+115·9	+121·9

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I may add that it is often a convenience to know at what rate the refraction in declination varies during an exposure. This may be computed in a similar manner. From the second of equations (α) we find

$$\Delta\delta = \Delta\zeta \cos\eta = \beta \tan\zeta \cos\eta,$$

or

$$\Delta\delta = \beta \, . \, \frac{\cos\delta \sin\phi - \sin\delta \cos\phi \cos h}{\sin\delta \sin\phi + \cos\delta \cos\phi \cos h}.$$

And since the apparent declination (δ') is equal to $\delta + \Delta\delta$, we have

$$\frac{d\delta'}{d\theta'} = \frac{d\Delta\delta}{d\theta} = \beta \, . \, \frac{\sin\phi \cos\phi \sin h}{\{\sin\delta \sin\phi + \cos\delta \cos\phi \cos h\}^2} \quad . \; . \; . \; . \quad (\gamma)$$

Hence we see that the apparent declination is always increasing on the west and always diminishing on the east of the meridian, as is otherwise obvious.

With the same substitution as before, we can write this expression in the form

$$\frac{d\delta'}{d\theta'} = \beta \, \frac{\cos\phi \, . \, \cos\nu}{\sin^2 n \sin^2(m+\delta)} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (\delta)$$

which is very convenient for computation.

This also varies within limits, which, in comparison with the accuracy of which measurements of stellar photographs are susceptible, may be considered wide. It is of course zero on the meridian, while at a zenith distance of 60° (at which it is possible to get first-rate photographs for measurement) it may amount to as much as 0″·5 per minute.

Dunsink Observatory:
1896 *December* 1.

Observations of the Leonids on 1896 *November* 13 *and* 14.

By Prof. Arthur A. Rambaut, M.A., D.Sc.

Although our observations this year at Dublin of the meteors belonging to the *Leonid* stream fall far short of what we hoped for, and in point of accuracy and completeness leave a good deal still to be desired, yet in view of the unfavourable character of the weather, which prevented observations at so many other stations, the results we obtained may possibly be of value, and ought at least to be put on record.

Cloudy weather prevented observations on the night of November 12.